3.8 What is a local linear approximation? Or a differential?

2014-11-05 day 49

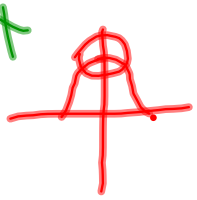
fire drill spot: 18C

If we were to approximate a function, like $f(x)=\cos x$, with a polynomials, around, say, $x=0$; Our polynomial would need to: → $P(x)$

1) $P(0)=\cos(0)=1$

2) $P'(0)=\frac{d}{dx}(\cos(x))\Big|_{x=0}=0$

3) the shape around $x=0$ of $P(x)$ must 'match' $\cos(x)$ around $x=0$



4) $P''(0)=\frac{d^2}{dx^2}(\cos x)\Big|_{x=0}=-\cos x\Big|_{x=0}=-1$

5) $P'''(0)=\frac{d^3}{dx^3}(\cos x)\Big|_{x=0}=+\sin x\Big|_{x=0}=0$

6) $P^{(4)}(0)=\frac{d^4}{dx^4}(\cos x)\Big|_{x=0}=+\cos x\Big|_{x=0}=+1$

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$-\frac{1}{\pi}x^2+1$

$-x^2+1$

x^2+1

$-x^2-x+1$

x^3+1

$-\frac{x^2}{2}+x+1$

$f(0)=1$

$f'(0)=0$ x

$f''(0)=-1$

$-2x^2+1$

$-4x$

-4

$f(0)$ ✓

$f'(0)$ ✓

$f''(0)$ x

$-\frac{1}{2}x^2+1$

$-x$

-1

$f(0)$ ✓

$f'(0)$ ✓

$f''(0)$ ✓

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"local" means in the neighborhood of a particular x.

Linear Approximation:

think approximating a f^n with a line.

in particular, the tangent line at a point.

"the" linear approximation to f(x) at $x=x_0$ needs

slope: $f'(x_0)$

point: $(x_0, f(x_0))$, i.e. (x_0, y_0)

we already know how to create the equation of this tangent line:

$$y - f(x_0) = f'(x_0)(x - x_0)$$

$$y = f(x) + f(x_0)$$

TaDa!! the local linear approx:

$$f(x) \approx y = f(x_0) + f'(x_0)(x - x_0)$$